Laboratoire Kastler Brossel
Physique quantique et applications
ENS
cnrs
SORBONNE UNIVERSITÉ

Continuous variable quantum networks -4

Valentina Parigi

Multimode quantum optics group

Continuous Variables Quantum Complex Networks team

Okinawa School in Physics: From quantum key distribution to the quantum internet (OSP2025)

September 21, 2025 - October 3, 2025

Non-Guassian elements in the network

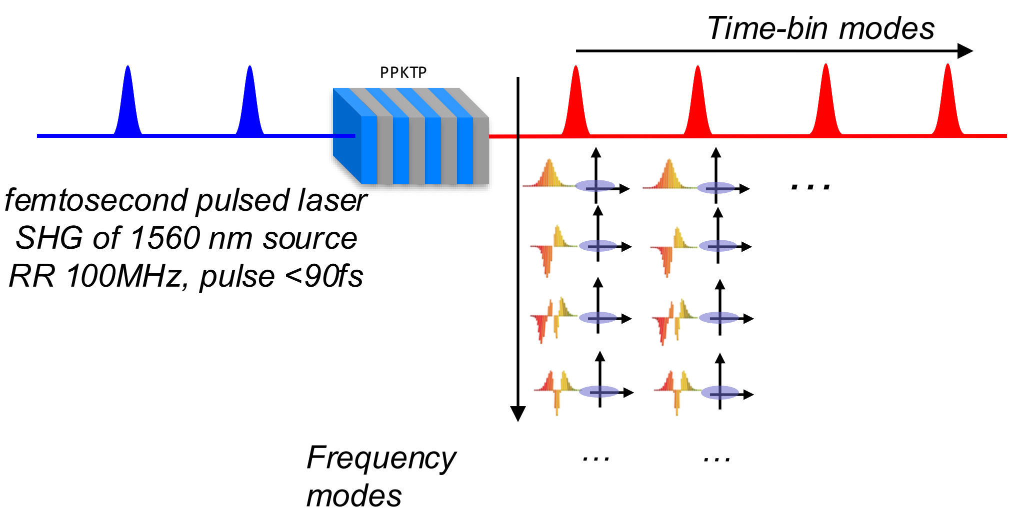
Multimode spectral structure

$$\hat{H} \propto \int d\lambda_S d\lambda_I \, f(\lambda_S, \lambda_I) \, \hat{a}^\dagger(\lambda_S) \hat{b}^\dagger(\lambda_I) + \text{h.c.}$$

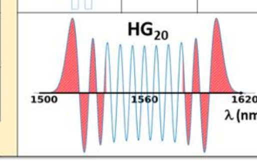
$$f(\lambda_S, \lambda_I) = p(\lambda_S, \lambda_I) \cdot \phi(\lambda_S, \lambda_I)$$

JSA *pump* *phase-matching*

Large number of involved modes:
merging strategy based on optical spectral shape with the one based on time-bin



Hermite-Gauss	Sqz	ASqz		Sqz	ASqz	Flat modes	Sqz	ASqz
0	-1.03	1.39	6	-0.73	1.45	0	-2.66	6.99
1	-0.68	1.31	7	-0.74	1.27	1	-2.43	6.49
2	-0.62	1.16	8	-0.54	1.16	2	-2.32	6.83
3	-0.61	1.27	12	-0.55	1.15	3	-2.01	6.47
4	-0.57	1.41	15	-0.60	1.36			
5	-0.58	1.46	20	-0.53	0.85			



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V. Roman-Rodriguez, B. Brecht, K. Srinivasan, C. Silberhorn, N. Treps, E. Diamanti, V. Parigi, *New J. Phys.* 23 043012 (2021)

V. Roman-Rodriguez, D. Fainsin, G. L. Zanin, N. Treps, E. Diamanti, and V. Parigi Physical Review Research 6, 043113 (2024)

Multimode spectral structure

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$$f(\lambda_S, \lambda_I) = p(\lambda_S, \lambda_I) \cdot \phi(\lambda_S, \lambda_I)$$

JSA *pump* *phase-matching*

Schmidt decomposition

$$f(\lambda_S, \lambda_I) = \sum_k \sqrt{r_k}\, u_k(\lambda_S) v_k(\lambda_I)$$

$$\hat{A}_k^\dagger = \int d\lambda\, u_k(\lambda)\hat{a}^\dagger(\lambda) \qquad \hat{B}_k^\dagger = \int d\lambda\, v_k(\lambda)\hat{b}^\dagger(\lambda)$$

They should be in the range of the pulse shaper of the LO!

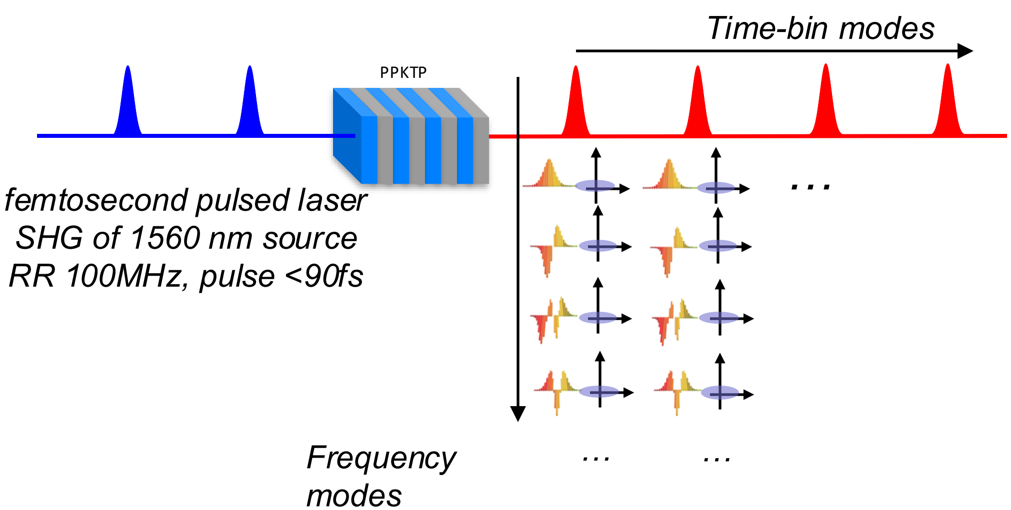
$$\hat{H} \propto \sum_k \sqrt{r_k}\, \hat{A}_k^\dagger \hat{B}_k^\dagger + \text{h.c.}$$

Type II

$$\hat{H} \propto \sum_k \sqrt{r_k}\, \hat{A}_k^{\dagger 2} + \text{h.c.}$$

Type 0

Large number of involved modes:
merging strategy based on optical spectral shape with the one based on time-bin



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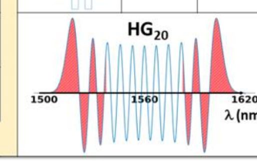

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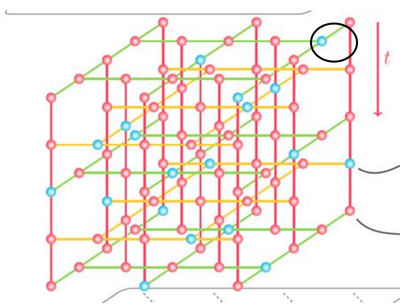
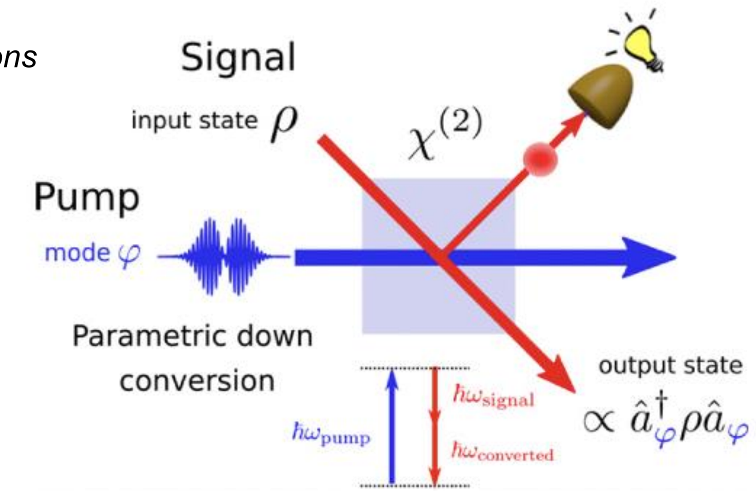
Mode-selective non Gaussian operation



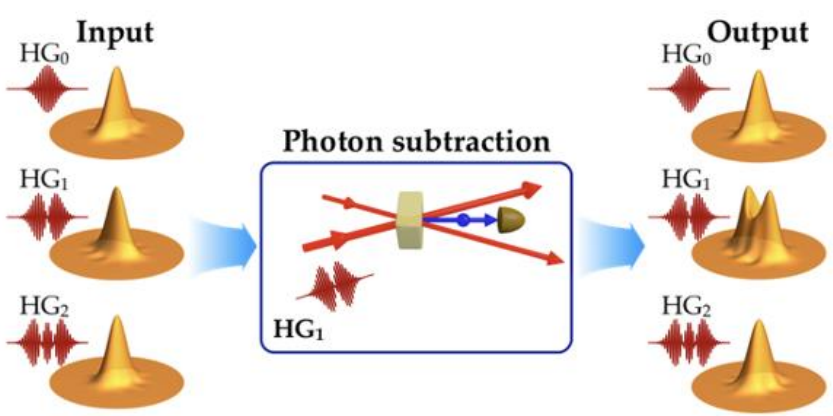
Spectral/time -selective operations

Mode-selective photon addition (parametric amplification)



G. Roeland et al New *J. Phys. 24 043031 (2022)*

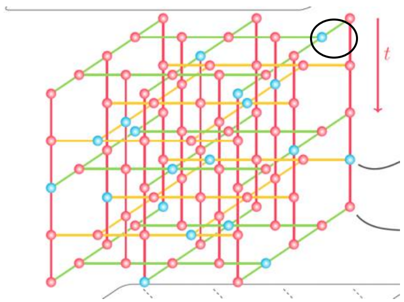
Mode-selective photon subtraction (Sum-frequency generation)



Y.-S. Ra, et al. Nature Physics 16, 144 (2020).
See also Quantum Pulse Gate (C. Silberhorn)

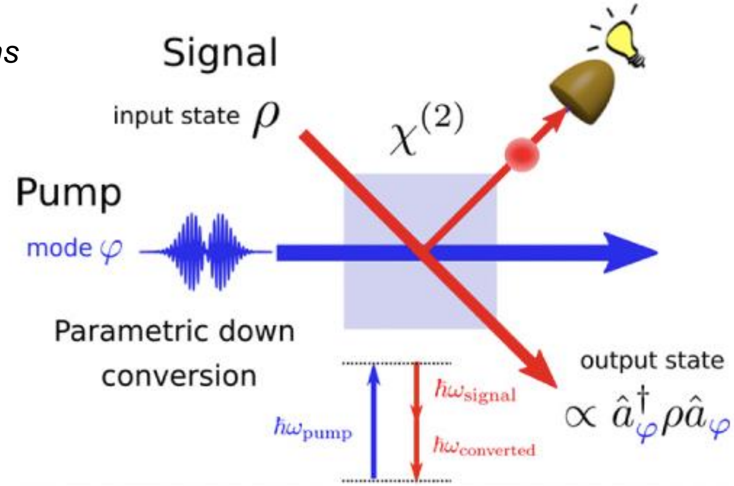
- Non-Gaussian statistics via linear and non-linear BS and heralding

Mode-selective non Gaussian operation



Spectral/time -selective operations

Mode-selective photon addition (parametric amplification)



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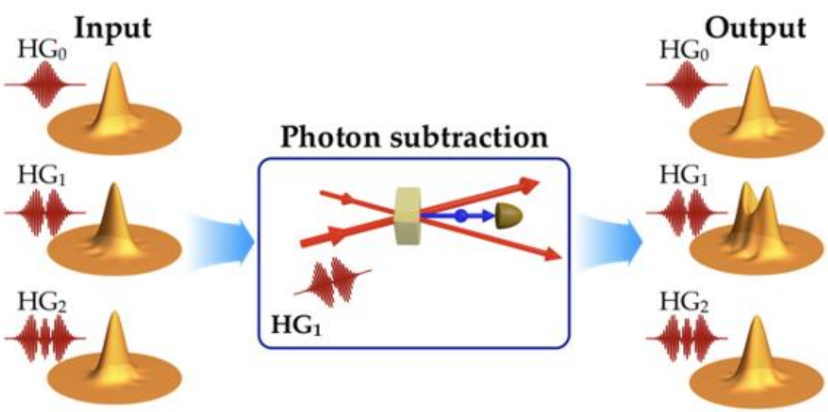
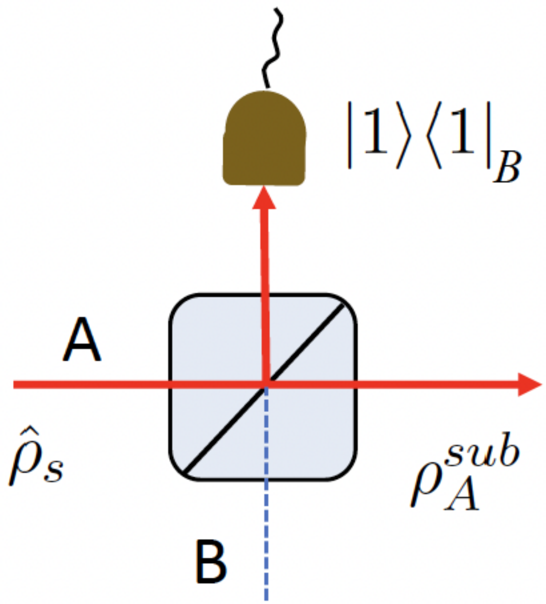


Y.-S. Ra, et al. Nature Physics 16, 144 (2020).
See also Quantum Pulse Gate (C. Silberhorn)

Engineering of non-linear optical processes (waveguides) collaboration Paderborn University C. Silberhorn

- Non-Gaussian statistics via linear and non-linear BS and heralding

You need a BS

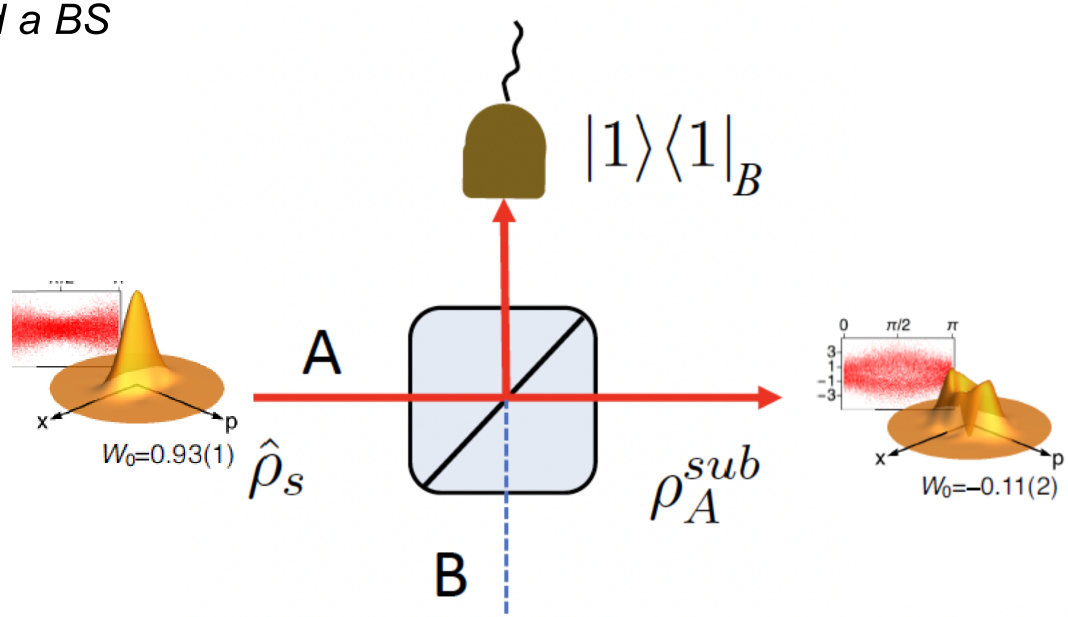


$$\rho_A^{sub} = \frac{\theta^2 \hat{a}\rho_A\hat{a}^\dagger}{Tr\{\theta^2\hat{a}\rho_A\hat{a}^\dagger\}} = \frac{\hat{a}\rho_A\hat{a}^\dagger}{Tr\{\hat{a}\rho_A\hat{a}^\dagger\}}$$

$$U(\theta) = Exp(\theta(\hat{b}^\dagger\hat{a} - \hat{b}\hat{a}^\dagger)) \approx \mathbb{1} + \theta(\hat{b}^\dagger\hat{a} - \hat{b}\hat{a}^\dagger)$$

Mode-selective single-photon subtraction

You need a BS



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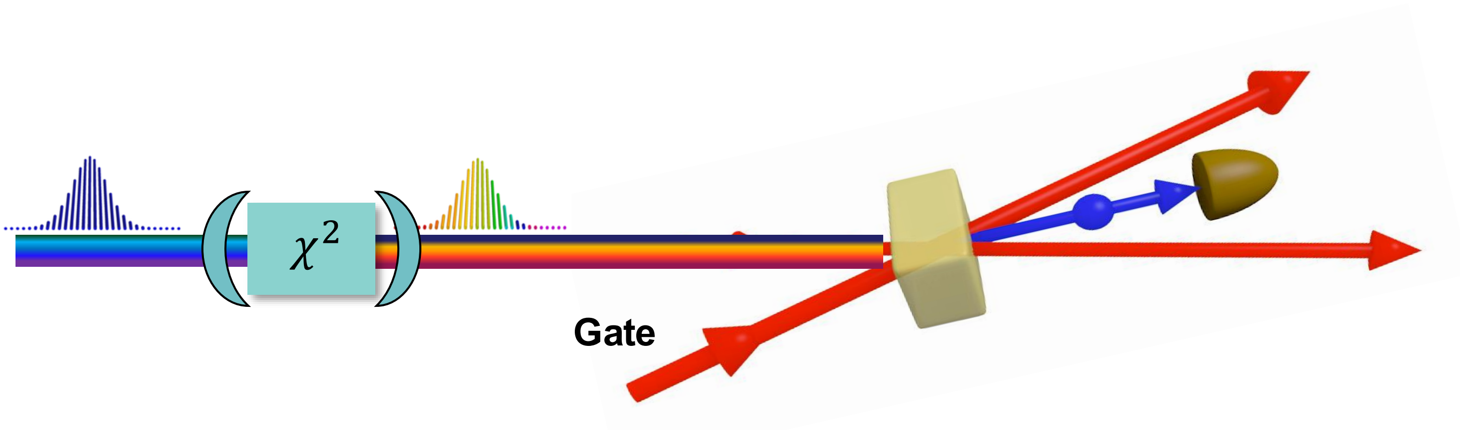
Mode-selective single-photon subtraction

You need a mode-selective BS



Sum frequency generation

You want the gate shape to control the mode from which the photon is subtracted

$$\hat{H} = \int\int d\omega_s d\omega_{\text{up}} T(\omega_s, \omega_{\text{up}}) \hat{a}(\omega_s) \hat{c}^\dagger(\omega_{\text{up}}) + \text{h.c.},$$

Gate

Transfer function

$$\begin{aligned} T(\omega_s, \omega_{\text{up}}) &= \alpha_g(\omega_{\text{up}} - \omega_s)\phi(\omega_s, \omega_{\text{up}}) \\ &= \sum_l \sqrt{\lambda_l} m_l(\omega_s) n_l(\omega_{\text{up}}) \end{aligned}$$

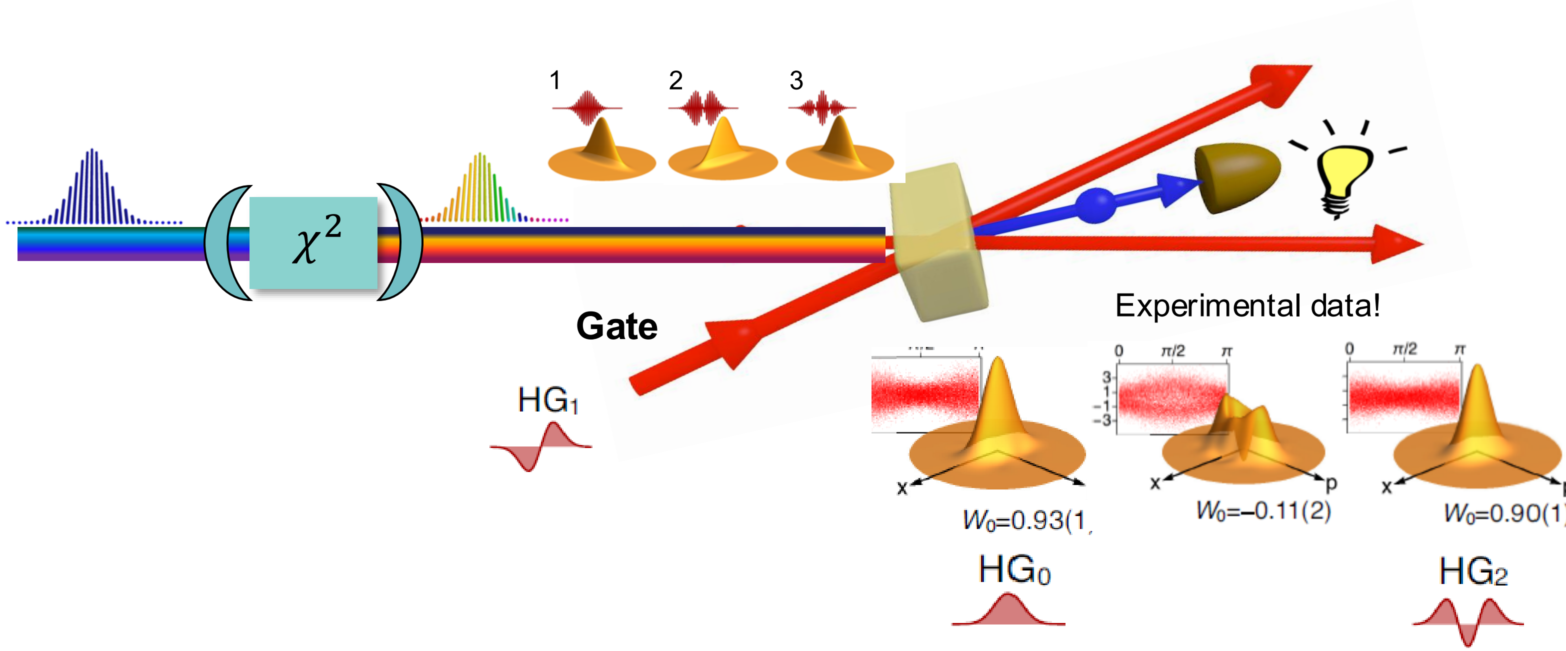
$$K = \frac{\left(\sum_n \lambda_n\right)^2}{\sum_n \lambda_n^2}$$

To get a pure state the process should be single-mode in the sense of Schmidt decomposition

Y.-S. Ra, et al. Nature Physics 16, 144 (2020).

Mode-selective single-photon subtraction

You need a mode-selective BS



Y.-S. Ra, et al. Nature Physics 16, 144 (2020);

Mode-selective single-photon subtraction

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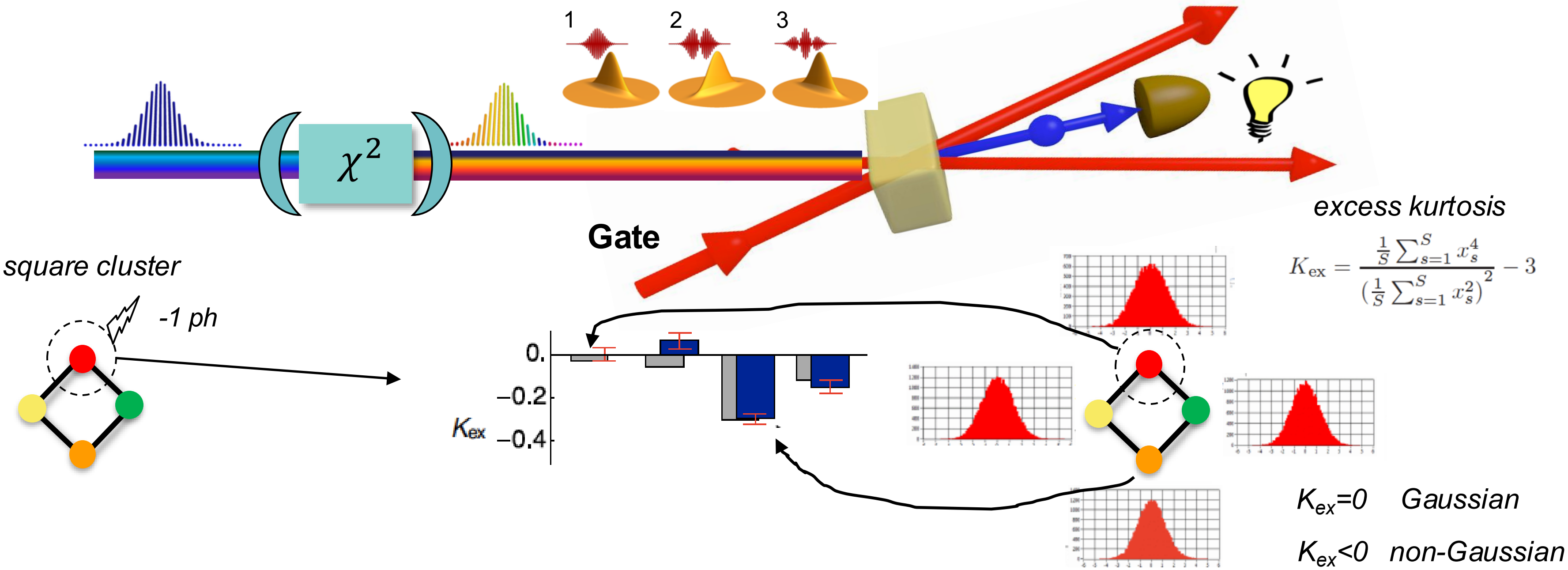

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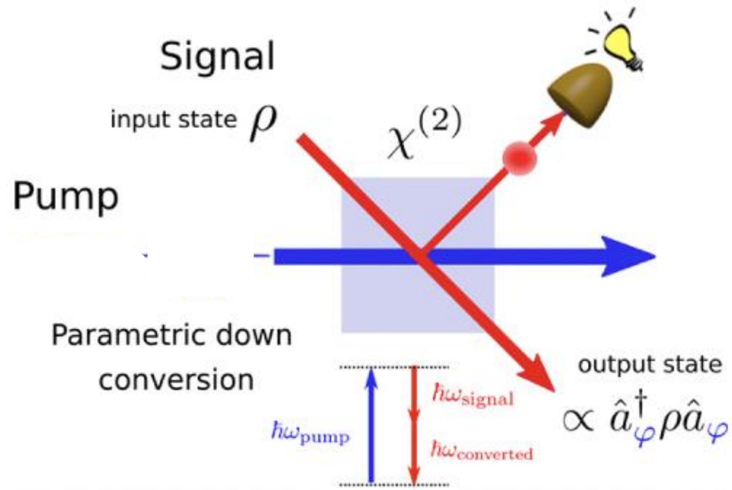
Mode-selective single-photon addition

You need a parametric process



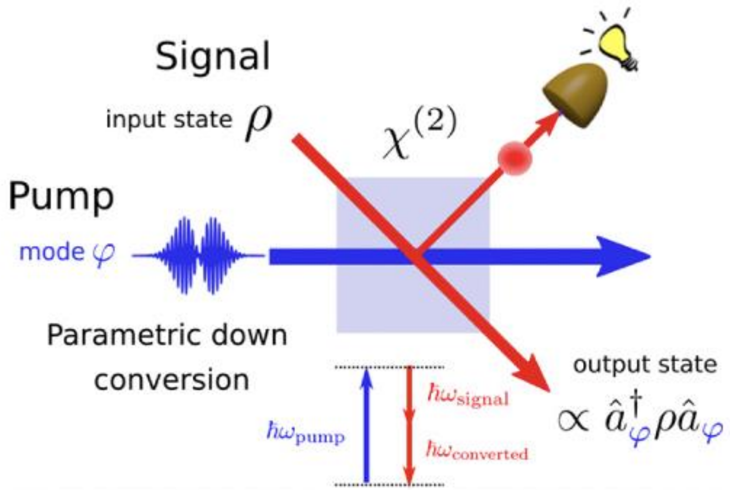
$$\rho_s^{add} = \frac{\lambda^2 \hat{a}^\dagger \rho_s \hat{a}}{Tr\{\lambda^2 \hat{a}^\dagger \rho_s \hat{a}\}} = \frac{\hat{a}^\dagger \rho_s \hat{a}}{Tr\{\hat{a}^\dagger \rho_s \hat{a}\}}$$

$$U(t) = Exp(-\frac{\imath H t}{\hbar}) \approx \mathbb{1} - \lambda(\hat{a}_s \hat{a}_i - \hat{a}_s^\dagger \hat{a}_i^\dagger)$$

G. Roeland et al New J. Phys. *24 043031 (2022), P Namdar et al, in preparation*

Mode-selective single-photon addition

You need a mode selective parametric process



You want the pump shape to control the mode to which the photon is added

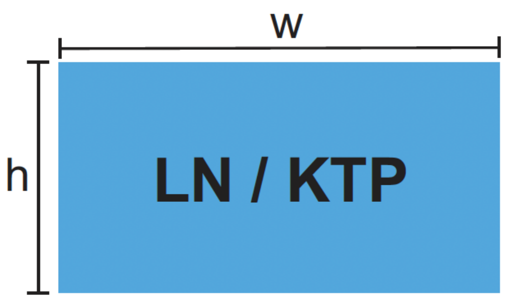
$$\hat{H} = \int\int d\omega_s d\omega_i \, J(\omega_s, \omega_i)\hat{a}^\dagger(\omega_s)\hat{b}^\dagger(\omega_i) + \text{h.c.}$$

$$J(\omega_s, \omega_i) = \alpha(\omega_s + \omega_i)\phi(\omega_s, \omega_i) = \sum_l \sqrt{\lambda_l} h_l(\omega_s) g_l(\omega_i)$$

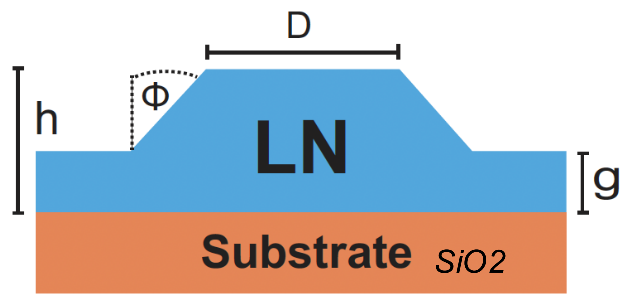
To get a pure state the process should be single-mode in the sense of Schmidt decomposition

Reverse engineering in wave-guides

Task: get the operations at telecom wavelength via diffusive or thin film waveguides



(a) Metallic waveguide geometry



(b) Thin-film waveguide geometry

1. *Precise modelling of the process*
2. *Run Evolutionary algorithm to get waveguide parameters. Fitness function= get K as close to 1 as possible !*

P. Namdar, P. Folge, C. E. Lopetegui, S. Babel, B. Brecht, C. Silberhorn, V. Parigi *Spectro-temporally tailored Non-Gaussian Quantum Operations in Thin-Film Waveguides*, arXiv:2508.04578


Peter Namdar,
Sorbonne University


Patrick F. Folge,
Paderborn Uiversity

Reverse engineering in wave-guides

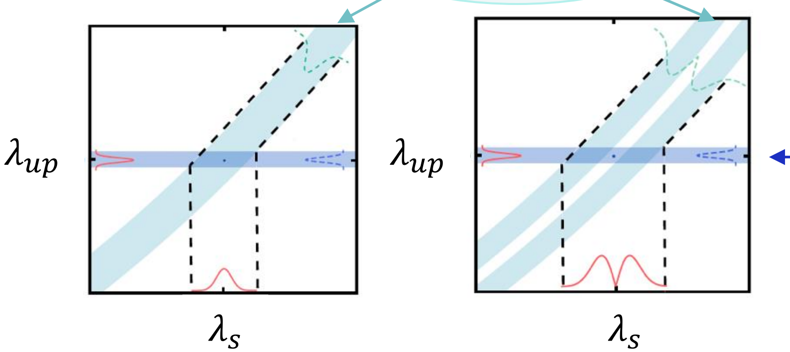
Task: get the operations at telecom wavelength via diffusive or thin film waveguides

Ideal case

Single-photon Subtraction

Transfer function

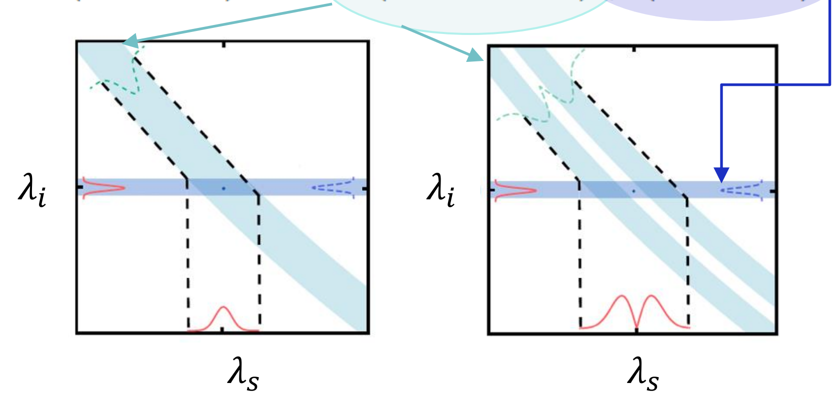
$$T(\omega_s,\omega_{\text{up}}) = \alpha_g(\omega_{\text{up}}-\omega_s)\phi(\omega_s,\omega_{\text{up}})$$



Single-photon Addition

Joint spectral amplitude

$$J(\omega_s,\omega_i) = \alpha(\omega_s+\omega_i)\phi(\omega_s,\omega_i)$$



No need to select the heralding mode (up or idler); the gate or pump mode are transferred into the signal

P. Namdar, et al. *Spectro-temporally tailored Non-Gaussian Quantum Operations in Thin-Film Waveguides*, arXiv:2508.04578

Reverse engineering in wave-guides

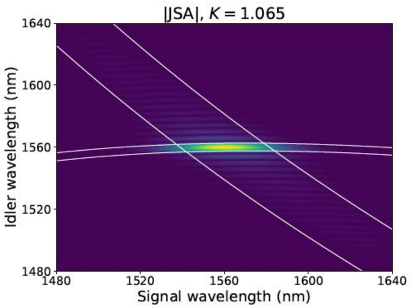
Task: get the operations at telecom wavelength via diffusive or thin film waveguides

Results

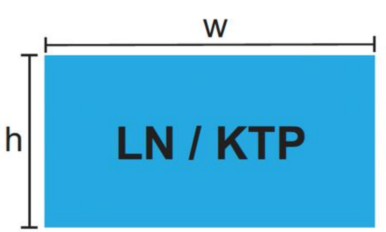
Single-Photon Addition

11

Parameter	Value
Length	7 mm
Width	2.8 μm
Height	2.3 μm
Crystal type	pp:KTP
Pump width	7.5 nm
Pump mode	HG 0/1/2
Type	II



(a) Pump mode: HG0



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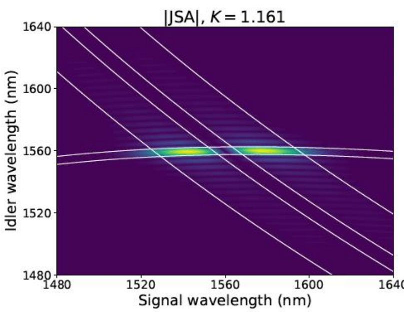
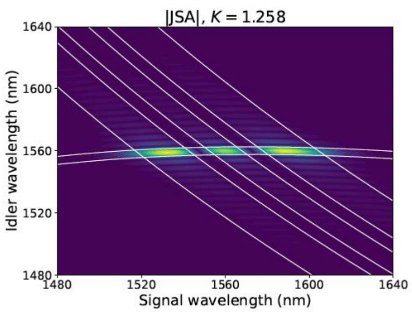


(b) Pump mode: HG1



(c) Pump mode: HG2

P. Namdar, et al. *Spectro-temporally tailored Non-Gaussian Quantum Operations in Thin-Film Waveguides*, arXiv:2508.04578

Reverse engineering in wave-guides

Task: get the operations at telecom wavelength via diffusive or thin film waveguides

Results

Single-Photon Addition

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Height	2.3 μm
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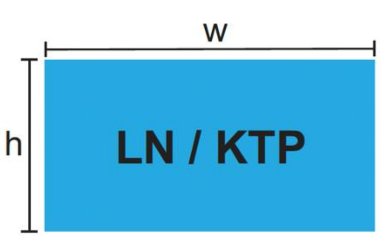
Pump width	7.5 nm
Pump mode	HG 0/1/2
Type	II



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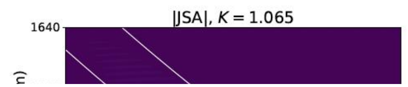
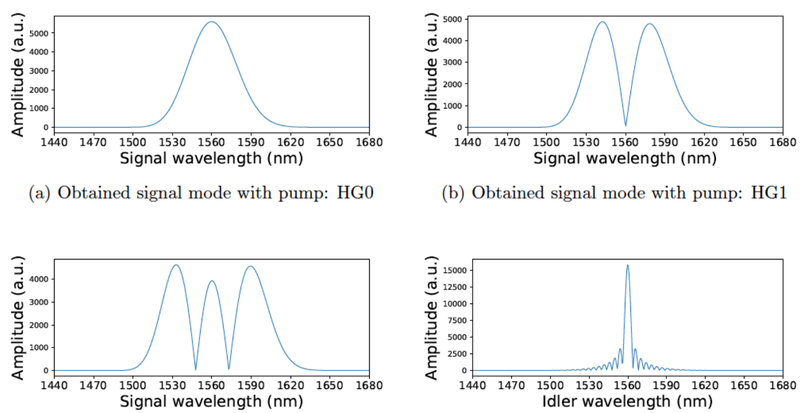


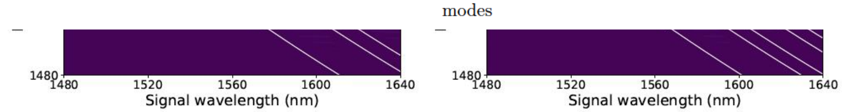


(a) Obtained signal mode with pump: HG0

(b) Obtained signal mode with pump: HG1

(c) Obtained signal mode with pump: HG2

(d) Obtained idler mode. Obtained with HG0-mode as pump. Representative for all pump-modes



(b) Pump mode: HG1

(c) Pump mode: HG2

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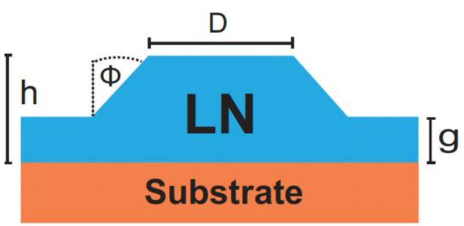
Reverse engineering in wave-guides

Task: get the operations at telecom wavelength via diffusive or thin film waveguides

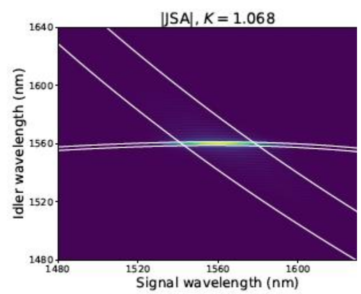
Results

Single-Photon Addition

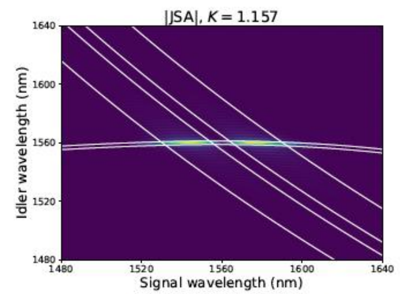
Parameter	Value
Length	7 mm
Width / D	1274 nm
Height / h	570 nm
Etching angle / ϕ	53.6°
Layer width / h - g	600 nm
Material	LN
Pump mode	HG 0/1/2
Type	II



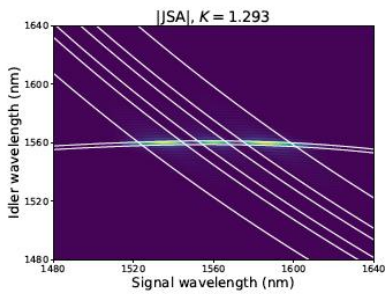
(b) Thin-film waveguide geometry



(a) Pump mode: HG0



(b) Pump mode: HG1



(c) Pump mode: HG2

P. Namdar, et al. *Spectro-temporally tailored Non-Gaussian Quantum Operations in Thin-Film Waveguides*, arXiv:2508.04578

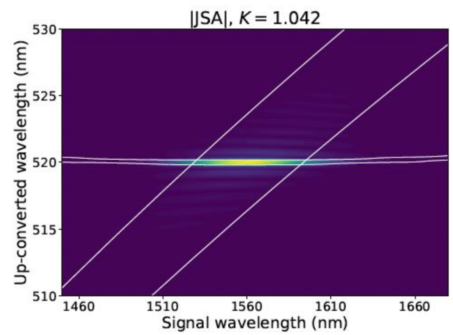
Reverse engineering in wave-guides

Task: get the operations at telecom wavelength via diffusive or thin film waveguides

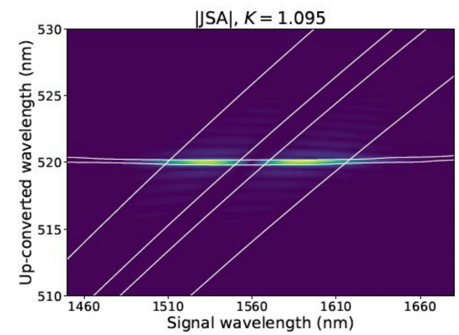
Results

Single-Photon Subtraction

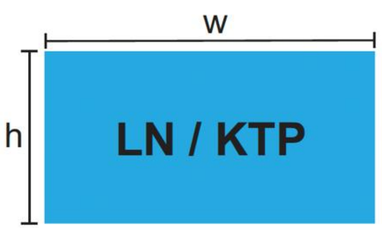
Parameter	Value
Length	2.0 mm
Width	2.5 μm
Height	1.8 μm
Material	KTP
Pump width	11.5 nm
Pump mode	HG 0/1/2
Type	0



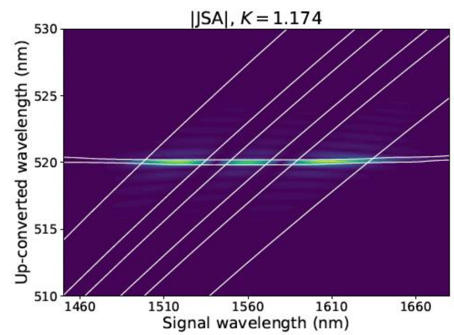
(a) Pump mode: HG0



(b) Pump mode: HG1



(a) Metallic waveguide geometry



(c) Pump mode: HG2

P. Namdar, et al. *Spectro-temporally tailored Non-Gaussian Quantum Operations in Thin-Film Waveguides*, arXiv:2508.04578

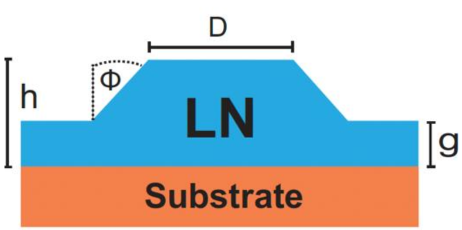
Reverse engineering in wave-guides

Task: get the operations at telecom wavelength via diffusive or thin film waveguides

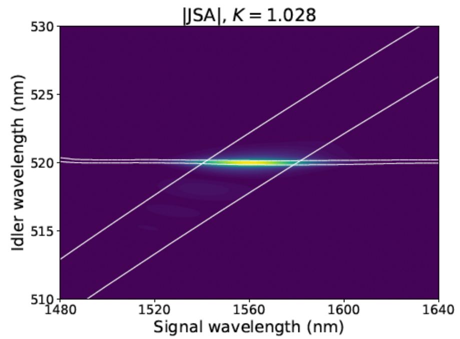
Results

Single-Photon Subtraction

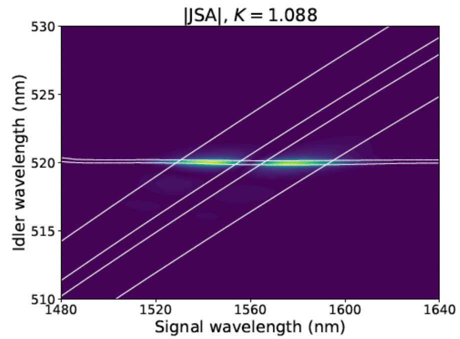
Parameter	Value
Length	7 mm
Width / D	883 nm
Height / h	774 nm
Etching angle / ϕ	75.2°
Layer width / h - g	788 nm
Material	LN
Pump width	7 mm
Pump mode	HG 0/1/2
Type	II



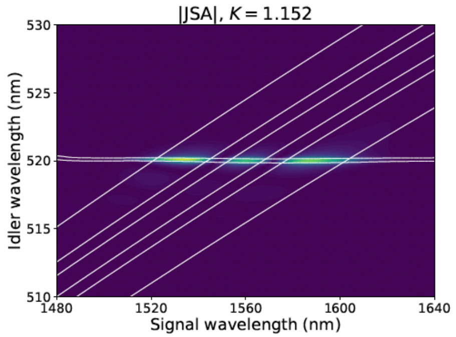
(b) Thin-film waveguide geometry



(a) Pump mode: HG0



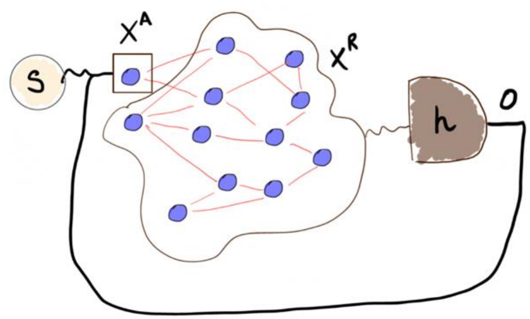
(b) Pump mode: HG1

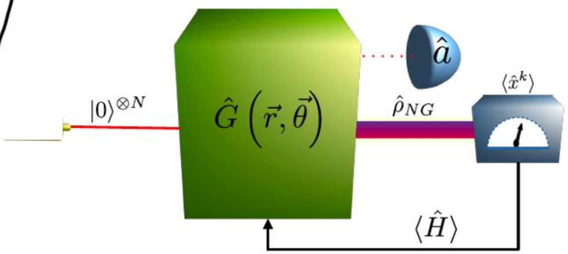


P. Namdar, et al. *Spectro-temporally tailored Non-Gaussian Quantum Operations in Thin-Film Waveguides*, arXiv:2508.04578

A reservoir, for local and distributed processes

Reservoir Computing, Variational Quantum algorithms





Collaboration Roberta Zambrini

J. Nokkala, R. Martínez-Peña, G. L. Giorgi, V. Parigi, M. C Soriano, R. Zambrini, Communications Physics volume 4, Article number: 53 (2021)

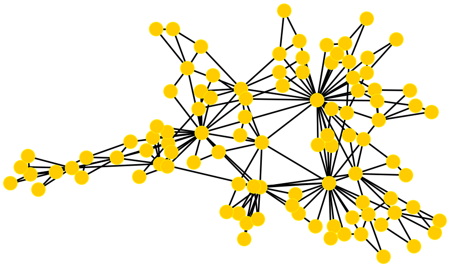
J Henaff, M Ansquer, MC Soriano, R Zambrini, N Treps, V Parigi arXiv:2401.14073 (2024), Optics Lett. Optics Letters 49, 2097 (2024)

J García-Beni, I Paparelle, V Parigi, GL Giorgi, MC Soriano, R Zambrini, EPJ Quantum Technology 12 (1), 1-14 (2025)

I. Paparelle, J. Henaff, J. Garcia-Beni, E. Gillet, G. L. Giorgi, M. C Soriano, R. Zambrini, V. Parigi arXiv arXiv:2506.07279 (2025)

P. Stornati *et al* Variational quantum simulation using non-Gaussian continuous-variable systems, accepted in Phys. Rev. Research 6, 043212 (2024)

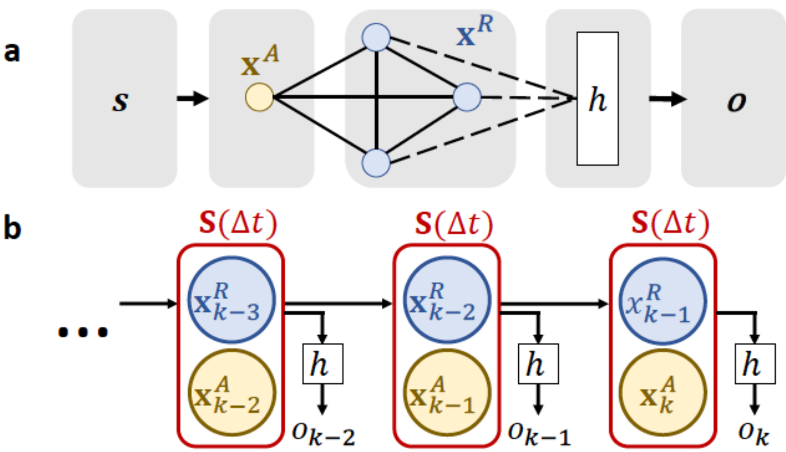
Reservoir Computing via Quantum states



Multimode Gaussian resources
+
Continuous Variable measurements

Johannes Nokkala

RC universal = when it can approximate any so-called fading memory function (continuous function of a finite number of past inputs)



reservoir space

$$\begin{cases} \mathbf{x}_k = T(\mathbf{x}_{k-1}, \mathbf{s}_k) \\ o_k = h(\mathbf{x}_k), \end{cases}$$

inputs

should correspond to a target function

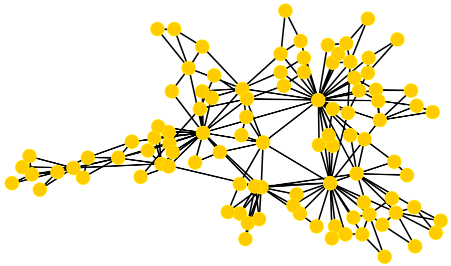
function of the features space

RC Universal in the case of Gaussian harmonic oscillators

required non-linearity-> tuned via encoding and output function

J. Nokkala, R. Martínez-Peña, G. L. Giorgi, V. Parigi, M. C Soriano, R. Zambrini, ***Gaussian states of continuous-variable quantum systems provide universal and versatile reservoir computing*** Communications Physics volume 4, 53 (2021)

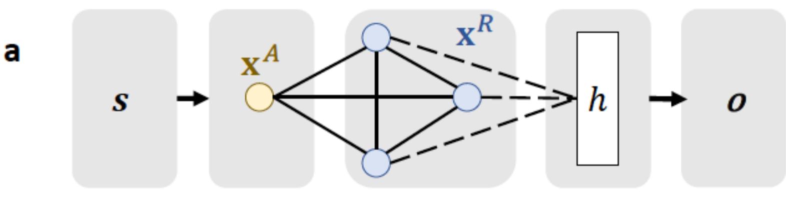
Reservoir Computing via Quantum states



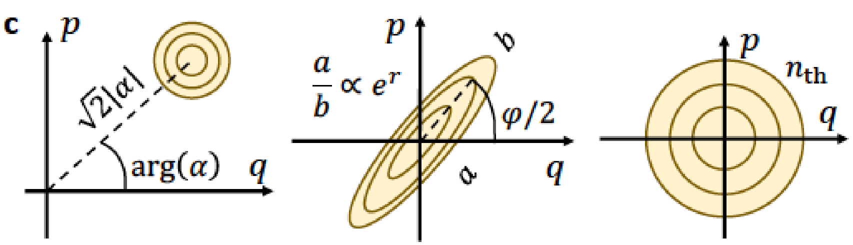
Multimode Gaussian resources
+
Continuous Variable measurements

Johannes Nokkala

quantum harmonic oscillators



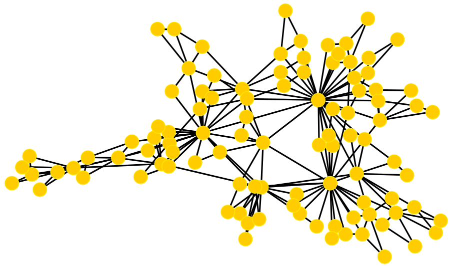
$$\begin{cases} \mathbf{x}_k = T(\mathbf{x}_{k-1}, \mathbf{s}_k) \\ o_k = h(\mathbf{x}_k), \end{cases}$$



coherent states *squeezed states* *thermal states*

J. Nokkala, R. Martínez-Peña, G. L. Giorgi, V. Parigi, M. C Soriano, R. Zambrini, ***Gaussian states of continuous-variable quantum systems provide universal and versatile reservoir computing*** Communications Physics volume 4, 53 (2021)

Reservoir Computing via Quantum states

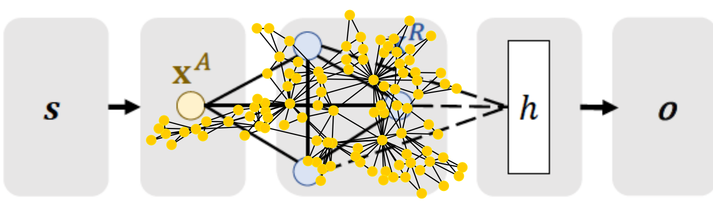


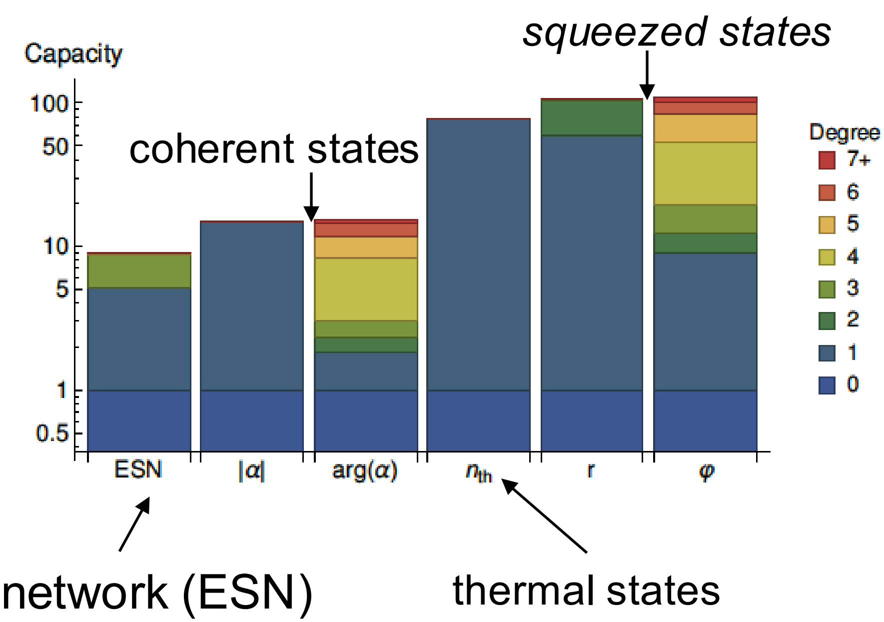
Multimode Gaussian resources
+
Continuous Variable measurements

Johannes Nokkala

quantum harmonic oscillators

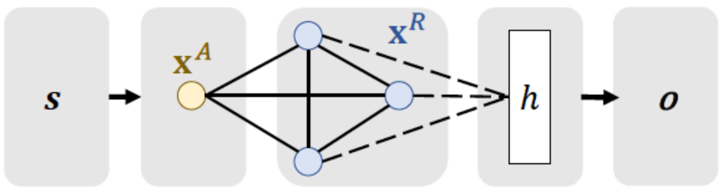
a





Measurement based CV Reservoir Computing

ii) letting the quantum reservoir evolve in time -Hamiltonian or master equation evolution- (reservoir layer)



The protocol is then repeated for the following input in the sequence

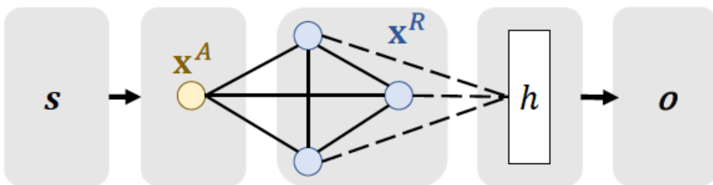
i) encoding the external input into the quantum substrate (input layer);

-state reset
-parametrized quantum circuit gates
-controlled driving into the Hamiltonian

(iii) measuring a set of observables to approximate a target task (readout layer –here linear regression is realized).

Measurement based CV Reservoir Computing

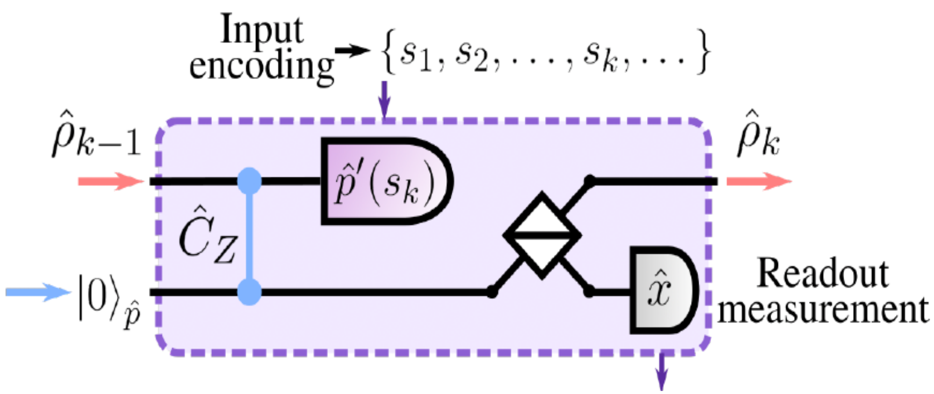
ii) letting the quantum reservoir evolve



i) encoding the external

(iii) measuring

Jorge García-Beni



-at each k-th time step, ***quantum teleportation*** *of the quantum reservoir state*

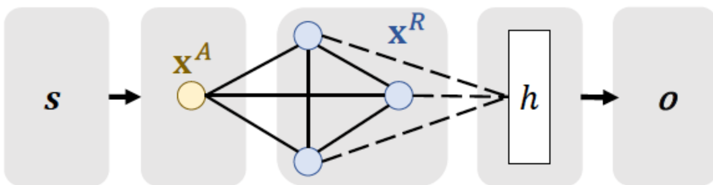
-The **input dependent quadrature is measured** *-> input dependent gate acts on the teleported (and new reservoir) state*

-Measurement via BS with vacuum to extract information

J García-Beni, I Paparelle, V Parigi, GL Giorgi, MC Soriano, R Zambrini, ***Quantum machine learning via continuous-variable cluster states and teleportation*** arXiv:2411.06907, accepted in EPJ Quantum Technology.

Measurement based CV Reservoir Computing

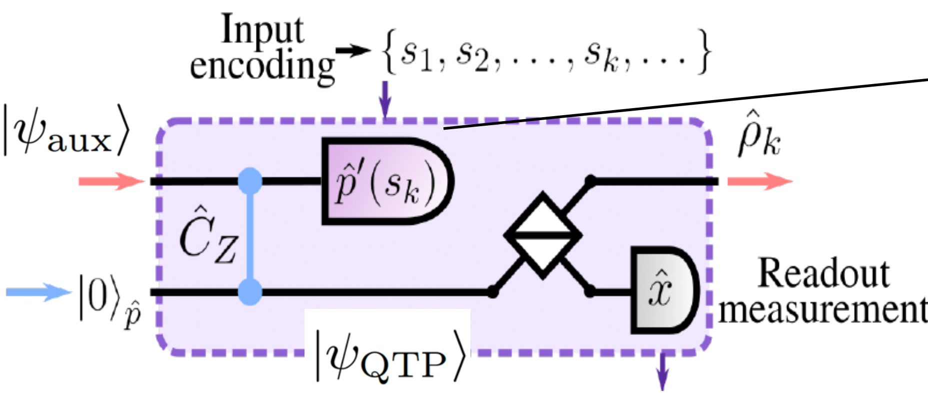
ii) letting the quantum reservoir evolve



i) encoding the external

(iii) measuring

Jorge García-Beni

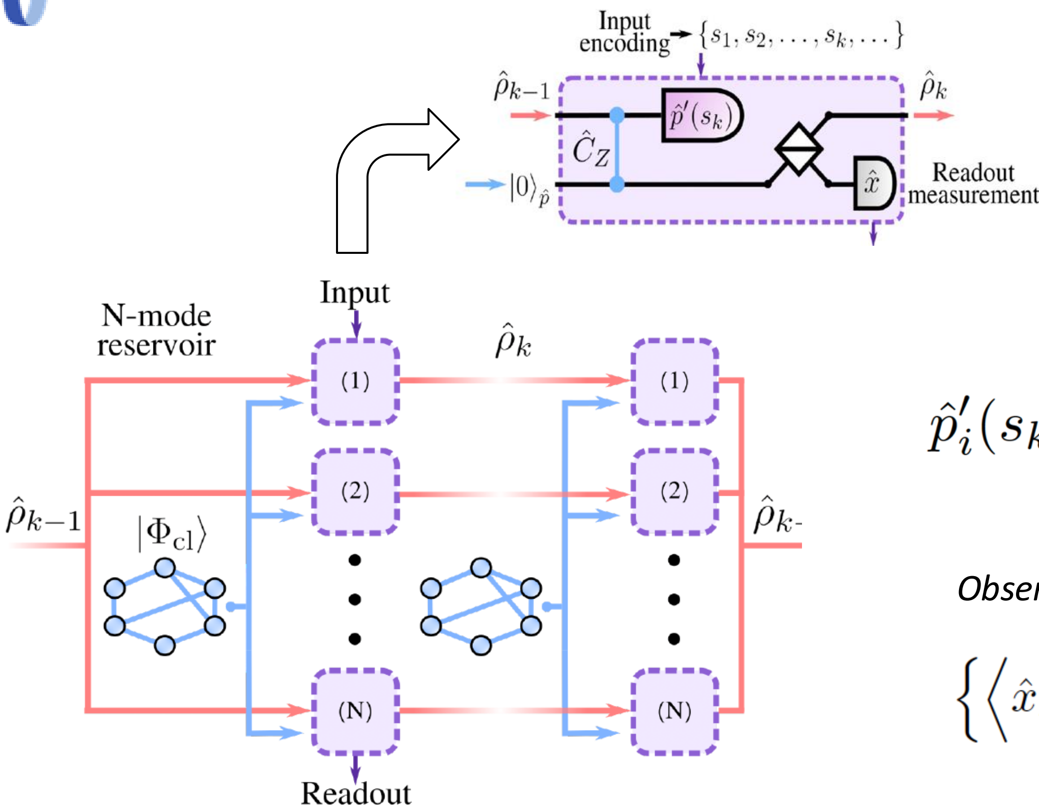


$$\hat{p}'(s) = \hat{p} + s\hat{x}$$

$$|\psi_{\text{QTP}}\rangle \propto e^{-is\hat{x}^2} |\psi_{\text{aux}}\rangle$$

J García-Beni, I Paparelle, V Parigi, GL Giorgi, MC Soriano, R Zambrini, ***Quantum machine learning via continuous-variable cluster states and teleportation*** arXiv:2411.06907, accepted in EPJ Quantum Technology.

Measurement based CV Reservoir Computing



mode-dependent parameters tuned according to the task

$$\hat{p}_i'(s_k) = \hat{p}_i + (\alpha_i s_k + \beta_i)\,\hat{x}_i$$

$$(i = 1, 2, \ldots N)$$

Observables

$$\left\{\left\langle \hat{x}_i^{(k)} \hat{x}_j^{(k)} \right\rangle\right\}_{i,j=1}^{N} \equiv \left\{O_l^{(k)}\right\}_{l=1}^{N(N+1)/2}$$

Linear regression to approach the target function $\bar{y}_k$

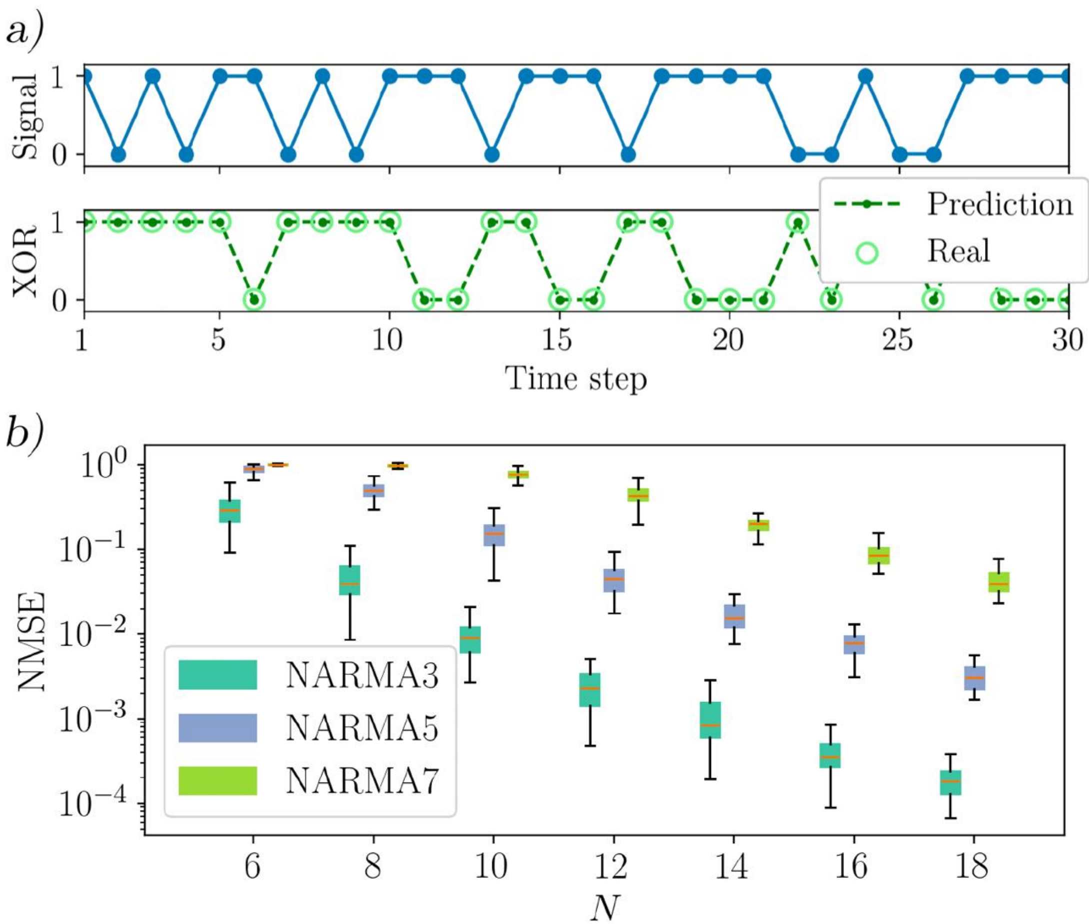
$$y_k = w_0 + \sum_{l=1}^{N(N+1)/2} w_l O_l^{(k)}$$

optimized

J García-Beni, I Paparelle, V Parigi, GL Giorgi, MC Soriano, R Zambrini, ***Quantum machine learning via continuous-variable cluster states and teleportation*** arXiv:2411.06907, accepted in EPJ Quantum Technology.

Measurement based CV Reservoir Computing

Tasks



Temporal XOR (100% accuracy)

it requires non-linearity and memory, 2 modes reservoir

NARMA -d

It requires both high memory and nonlinearity

$$\bar{y}_k^{(d)} = \alpha \bar{y}_{k-1} + \beta \bar{y}_{k-1} \sum_{i=1}^{d} \bar{y}_{k-i} + \gamma u_{k-1} u_{k-d} + \delta$$

Experimental CV quantum Reservoir Computing

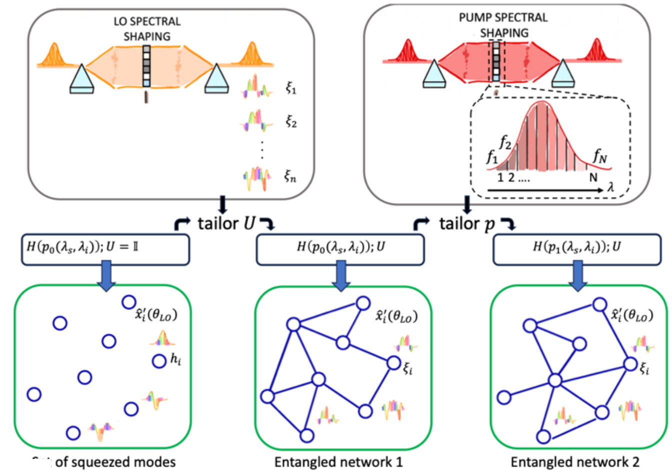
Experimental memory control in continuous variable optical quantum reservoir computing

Iris Paparelle,[1] Johan Henaff,[1] Jorge García-Beni,[2] Émilie Gillet,[1] Gian Luca Giorgi,[2] Miguel C. Soriano,[2] Roberta Zambrini,[2] and Valentina Parigi[1,*]

[1] *Laboratoire Kastler Brossel, Sorbonne Université, ENS-Université PSL, CNRS, Collège de France, 4 place Jussieu, 75252 Paris, France*

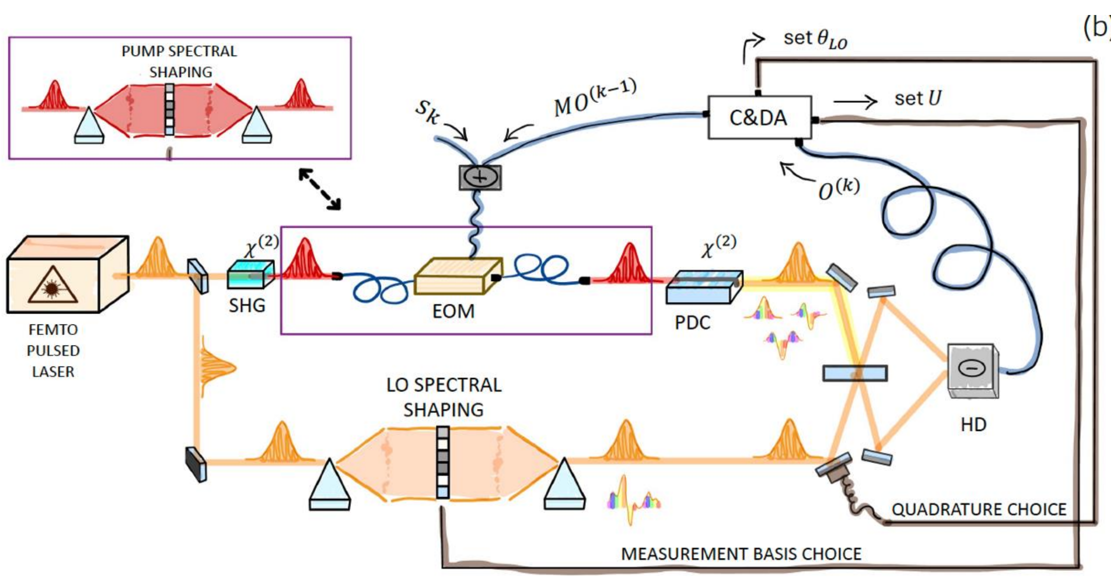
[2] *Instituto de Física Interdisciplinar y Sistemas Complejos (IFISC), UIB–CSIC UIB Campus, Palma de Mallorca, E-07122, Spain*

(Dated: June 10, 2025)



$$\begin{cases} \mathbf{x}_k = T(\mathbf{x}_{k-1}, \mathbf{s}_k) \\ o_k = h(\mathbf{x}_k), \end{cases}$$

$$\begin{cases} O_m^{(k)} \propto \sigma'^{(k)}_{ij} \mid \sigma'^{(k)} = S_U{}^T\, \sigma(H(p(s_k, O^{(k-1)})))\, S_U \\ y_k = \mathbf{w}^T \mathbf{O}^{(k)} + b \end{cases}$$



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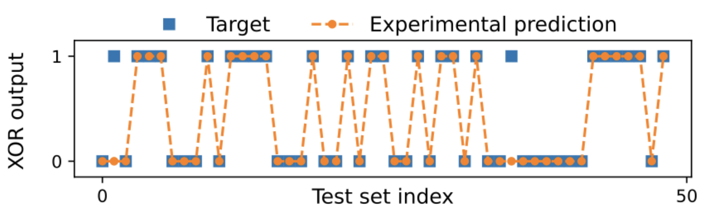

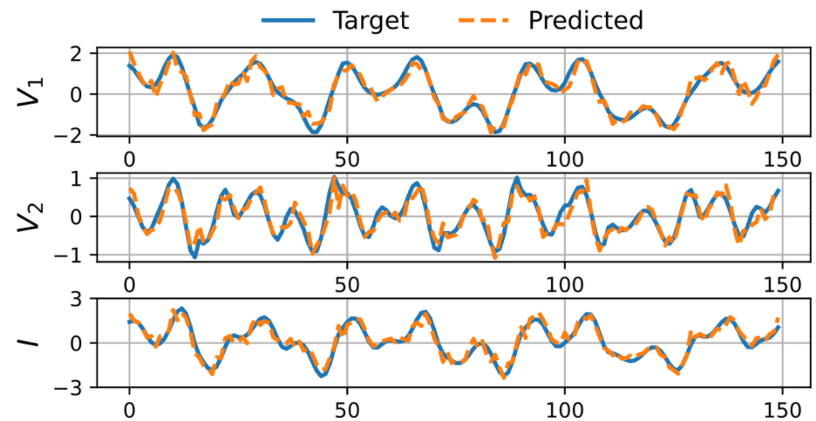

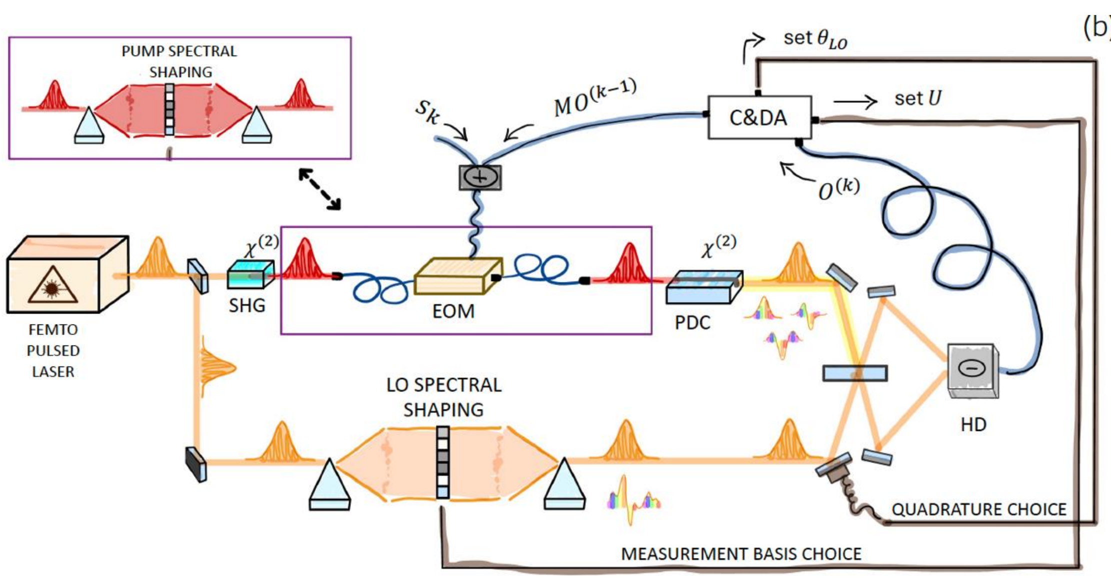

Experimental CV quantum Reservoir Computing

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